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Vibrator Data Denoising Based on Fractional Wavelet Transform

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Abstract

In this paper, a novel data denoising method is proposed for seismic exploration with a vibrator which produces a chirp-like signal. The method is based on fractional wavelet transform (FRWT), which is similar to the fractional Fourier transform (FRFT). It can represent signals in the fractional domain, and has the advantages of multi-resolution analysis as the wavelet transform (WT). The fractional wavelet transform can process the reflective chirp signal as pulse seismic signal and decompose it into multi-resolution domain to denoise. Compared with other methods, FRWT can offer wavelet transform for signal analysis in the time-fractional-frequency plane which is suitable for processing vibratory seismic data. It can not only achieve better denoising performance, but also improve the quality and continuity of the reflection syncphase axis.

Key words: seismic method, vibrator data, chirp signals, noise attenuation, fractional wavelet transform.

1. INTRODUCTION

The methods generally called vibratory seismic have been very important in seismic exploration. Many works focus on the research of highly efficient acquisition methods of vibroseis data, such as cascaded sweeps, slip-sweep acquisition, simultaneous shooting, and so on (Bagaini 2010). With the development of the exploration methods, the corresponding signal processing methods have been drawing more and more attention (Jeffryes 1996, Sallas *et al.* 1998, Meunier and Bianchi 2002), since the seismic noise shrinkage becomes more and more important, as the requirements of high-quality data for post-processing increase. Transform-based methods are the most popular. If the seismic data is spare in the transform domain, denoising goal can be achieved by eliminating the small coefficients which are considered to be noise. The vibratory seismic system produces a chirp-like signal, which makes it difficult to find an optimum transform domain in which seismic data can be sparsely represented.

To date, some of the recently developed signal processing methods are introduced into this field, *e.g.*, wavelet transform (WT), fractional Fourier transform (FRFT), Radon-Wigner transform, *etc*. However, they have different disadvantages for chirp-like signals processing. The WT, which enables a researcher to study features of the signal locally with a detail matched to their scale (Morlet *et al.* 1982), *i.e.*, broad features on a large scale and fine features on a small scale, may not able to offer the sparest representation of the signal. The FRFT, which can represent signal in the time fractional frequency domain, is a global transformation, so it limits the usage on the area of analyzing signal's localized characteristics (Miah and Sacchi 2011). The Radon-Wigner transform (Wood and Barry 1994) can analyze local property of chirp-like signals (Steeghs 1998), but it has cross-term problem with the quadratic time-frequency representation.

Mendlovic *et al.* (1997) first proposed the FRWT to analyze optical signals. Then, Huang and Suter (1998) proposed the fractional wave packet transform (FRWPT). Shi *et al.* (2012) proposed a novel fractional wavelet transform (NFRWT) based on fractional convolution theorem. There are some other works which define fractional wavelet transform based on the fractional splines (Unser and Blu 2000). In this work, the FRWT is introduced to suppress noise in the vibratory seismic system. As a generalization of WT, the FRWT combines the advantages of the wavelet transform and the FRFT; it is a linear transform without cross-term interference and is capable of providing multiresolution analysis and representing signals in the fractional domain. Thus, it is more suitable to analyze chirp-like signals.

The rest of the paper is organized as follows. In Section 2, we briefly introduce the characteristics of linear frequency modulation (LFM) signal (chirp-like signal which is used as a vibratory source). In Section 3, we propose the FRWT by treating the wavelet transform as a filter defined in fractional domain. Then the FRWT is applied to analyze the reshaped vibratory seismic signal. The performance is analyzed and compared with other signal processing methods in Section 4. Finally, conclusions are provided in Section 5.

2. LINEAR FREQUENCY MODULATION SIGNAL

Linear frequency modulation (LFM) signal is a common kind of chirp-like signal, which is widely used in radar and vibratory seismic systems:

$$s_t(t) = A\,\mathrm{rect}\left(\frac{t}{T}\right)e^{j2\pi(f_0 t + \frac{1}{2}\beta t^2)}\ , \tag{1}$$

where f_0 is the carried frequency, A is the amplitude, $\beta = B/T$ is the frequency rate, B is the bandwidth, T is the pulse width, rect(w) is a rectangular pulse with width w. The waveform of the signal is shown in Fig. 1A.

Instantaneous frequency of the signal can be expressed as:

$$f(t) = \frac{1}{2\pi}\frac{d}{dt}\left(2\pi\left(f_0 t + \frac{1}{2}\beta t^2\right)\right) = f_0 + \beta t\ . \tag{2}$$

A linear relationship, as shown in Fig. 1B, exists between the instantaneous frequency and time.

The frequency spectrum of chirp signal can be written as:

$$S(f) = A\sqrt{\frac{\pi}{\beta}}\left\{\left[c(u_1)+c(u_2)\right]^2 + \left[s(u_1)+s(u_2)\right]^2\right\}^{1/2} e^{j\theta(f)}\ , \tag{3}$$

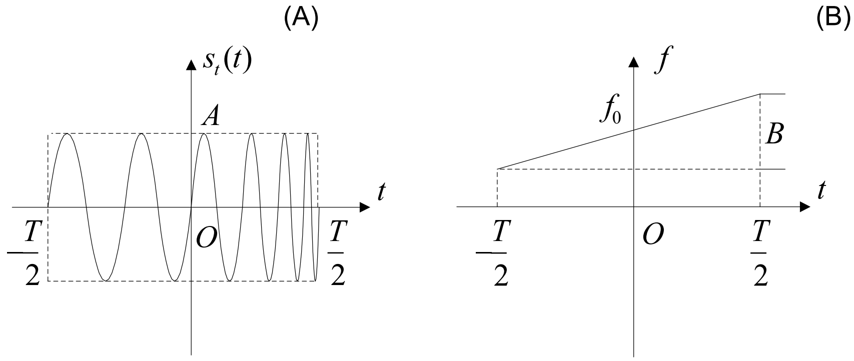

Fig. 1. Linear frequency modulation signals: (A) chirp waveform, and (B) instantaneous frequency.

where

$$\theta(f) = -\frac{2\pi^2}{\beta}\left(f - f_0\right)^2 + \arctan\left[\frac{s(u_1)+s(u_2)}{c(u_1)+c(u_2)}\right], \tag{4}$$

$$u_1 = \sqrt{D}\,\frac{1+2(f-f_0)/\Delta f}{\sqrt{2}}, \tag{5}$$

$$u_2 = \sqrt{D}\,\frac{1-2(f-f_0)/\Delta f}{\sqrt{2}}, \tag{6}$$

$$c(u) = \int_0^u \cos\left(\frac{\pi}{2}x^2\right)dx, \tag{7}$$

$$s(u) = \int_0^u \sin\left(\frac{\pi}{2}x^2\right)dx, \tag{8}$$

where $D = BT$, T is the width of pulse, and B is the width of spectrum.

3. THEORY

3.1 Fractional Fourier transform (FRFT)

Fractional Fourier transform (FRFT), which is widely used in non-stationary signal processing, is a generalized form of Fourier transform. FRFT can be considered as a rotation operator in the time-frequency plane.
The kernel of continuous FRFT is defined as:

$$K_\alpha(u,t) = \begin{cases} A_\alpha e^{j\pi\left[(t^2+u^2)\cot\alpha - 2ut\csc\alpha\right]}, & \alpha \neq n\pi \\ \delta\left[t-(-1)^n u\right], & \alpha = n\pi \end{cases}, \tag{9}$$

where α can be considered as an angle with the time axis,

$$A_\alpha = \sqrt{1 - j\cot\alpha}\ .$$

When $\alpha = \pi/2$ in Eq. 1 one obtains the traditional Fourier transform.
The FRFT and inverse FRFT (IFRFT) of the signal $x(t)$ are:

$$\begin{aligned} X_\alpha(u) &= \left\{F_\alpha[x(t)]\right\}(u) = \int_{-\infty}^{+\infty} K_\alpha(u,t)x(t)dt, \\ x(t) &= \left\{F_{-\alpha}[X_\alpha(u)\right\}(t) = \int_{-\infty}^{+\infty} K_{-\alpha}(t,u)X_\alpha(u)du, \end{aligned} \tag{10}$$

where $F_\alpha[\bullet]$ is the operator of FRFT.

The calculation of FRFT can be divided into the following three steps:

1. Multiplying LFM signal:

$$g(t) = e^{j\pi t^2 \cot\alpha} x(t) \;;$$

2. Calculating FRFT of $g(t)$, the argument's scaling is included:

$$X'_\alpha(u) = \int_{-\infty}^{+\infty} e^{j2\pi ut\csc\alpha} g(t)dt \;;$$

3. Multiplying LFM signal:

$$X_\alpha(u) = A_\alpha e^{j\pi u^2 \cot\alpha} X'_\alpha(u) \;.$$

It is obvious that the existing condition of FRFT is consistent with FFT. The algorithm only needs two chirp multiplications and one FFT calculation. It meets the analysis and synthesis requirements for the signal because the reversibility and periodicity have been demonstrated by Kraniauskas *et al.* (1998), Ozaktas *et al.* (1996), and Pei and Ding (2000).

3.2 Novel fractional wavelet transform (FRWT)

The FRFT has been widely used in signal procession field in recent years. However, the FRFT tells us the fractional frequencies which exist across the whole duration of the signal but not the frequencies which exist only at a particular time. It means that the FRFT is a global transform and it fails in obtaining local information of the signal, which is crucial for processing non-stationary signals. With the knowledge that the wavelet transform is a localized transformation and is efficient for transient signal processing, a generalized wavelet transform, called fractional wavelet transform, which can offer signal representation in the time-fractional-frequency plane, has been developed; it inherits both the advantages of multi-resolution analysis of the wavelet transform and the capability of the signal representation in the fractional domain which is similar to FRFT. A general deduction of FRWT can be found below.

The wavelet transform can be defined as a convolution in the time domain:

$$W(a,b) = \left\langle f(t), \varphi_{a,b}(t)\right\rangle = \int_0^\infty f(t)\varphi_{a,b}^*(t)dt = \int_0^\infty f(t)\frac{1}{\sqrt{a}}\varphi^*\left(\frac{t-b}{a}\right)dt = f(t)\otimes\left(\frac{1}{\sqrt{a}}\varphi^*\left(\frac{-t}{a}\right)\right), \quad (11)$$

where $\varphi(t)$ is the mother wavelet, a and b are the parameters which are used to control dilation and translation, respectively.

As we know, the fractional convolution can be expressed as:

$$f(t)\underset{\alpha}{\otimes}\left(h(t)\right) = A_\alpha e^{-j\pi t^2\cot\alpha}\cdot\left[\left(f(t)e^{j\pi t^2\cot\alpha}\right)\otimes\left(h(t)e^{j\pi t^2\cot\alpha}\right)\right] = e^{-j\pi t^2\cot\alpha}\cdot\left\langle f(\cdot)e^{j\pi(\cdot)^2\cot\alpha}, h^*(t-\cdot)e^{-j\pi(t-\cdot)^2\cot\alpha}\right\rangle. \quad (12)$$

The output in the fractional Fourier domain is:

$$F_\alpha\left[f(t)\underset{\alpha}{\otimes}\left(h(t)\right)\right]=F_\alpha(u)H(u\csc\alpha)e^{-j\pi\cot\alpha u^2}\ . \tag{13}$$

Applying the fractional convolution to the LFM signal which is introduced by vibratory seismic systems, we can find that the item of frequency modulation can be demodulated by a chirp signal. The chirp item in Eq. 13 is not necessary for the analysis. If we modify the convolution, we can get a simplified expression in the fractional Fourier domain without the chirp signal term:

$$F_\alpha\left[f(t)\underset{\alpha}{\otimes}\left(h(t)e^{-j\pi\cot\alpha t^2}\right)\right]=F_\alpha(u)H(u\csc\alpha) \tag{14}$$

extends the wavelet transform with the fractional convolution expressed as:

$$W_\alpha(a,b)=f(t)\underset{\alpha}{\otimes}\left(\frac{1}{\sqrt{a}}\varphi^*\left(\frac{-t}{a}\right)e^{-j\pi t^2\cot\alpha}\right)=e^{-j\pi b^2\cot\alpha}\left\{\left(f(t)e^{j\pi t^2\cot\alpha}\right)\otimes\left(\frac{1}{\sqrt{a}}\varphi^*\left(\frac{-t}{a}\right)e^{-j\pi t^2\cot\alpha}e^{j\pi t^2\cot\alpha}\right)\right\}$$

$$=\int_{-\infty}^{\infty}f(t)e^{j\pi(t^2-b^2)\cot\alpha}\varphi_{a,b}^*(t)dt=\left\langle f(\cdot),\varphi_{a,b}(\cdot)e^{-j\pi(\cdot)^2\cot\alpha}e^{j\pi b^2\cot\alpha}\right\rangle=\left\langle f(\cdot),\varphi_{\alpha,a,b}(\cdot)\right\rangle\ . \tag{15}$$

where $W_\alpha(a,b)$ expresses the fractional wavelet transform:

$$\varphi_{\alpha,a,b}(t)=\varphi_{a,b}(t)e^{-j\pi(t^2-b^2)\cot\alpha}\ .$$

The inverse transform can be calculated as follows:

$$f(t)=\frac{1}{C_{\alpha,\varphi}}\int_0^{\infty}\int_{-\infty}^{\infty}W_\alpha(a,b)\varphi_{\alpha,a,b}(t)\,db\frac{da}{a^2}\ , \tag{16}$$

where $C_{\alpha,\varphi}$ is a constant that depends on the wavelet used. The success of the reconstruction depends on this constant, called the admissibility constant; it should satisfy the following admissibility condition:

$$C_{\alpha,\varphi}=\int_0^{\infty}\frac{\left|\Psi(u\csc\alpha)\right|^2}{u}du<\infty\ , \tag{17}$$

where $\Psi(x)$ denotes the Fourier transform of $\varphi(t)$. The admissibility condition implies that $\Psi(0) = 0$, which is $\int_{-\infty}^{\infty}\varphi(t)dt=0$. Consequently, continuous fractional wavelets must oscillate and behave as bandpass filters in the fractional Fourier domain. Whenever $\alpha = \pi/2$, the FRWT reduces to the classical wavelet transform.

4. EXPERIMENT

The wavelet transform is actually a differently scaled bandpass filter in frequency domain, as we can see from Section 3.3. If the signal's energy is well concentrated in the frequency domain, the wavelet denoising methods is one of the most effective ways to suppress noise. As for the signals whose energy is not well concentrated in the frequency domain, *e.g.*, chirp signals, the signals' energy may be well focused in a certain fractional domain, so it is suitable for FRWT to process this kind of signals.

The computation of the fractional wavelet transform for the vibratory seismic signals can be done in the following steps:

(i) Multiply a chirp signal and do wavelet transform. The process of multiplying a chirp signal can help to modify the concentrated performance of the signal energy. After the wavelet transform, signals are decomposed into different fractional-frequency bands with different scales.

(ii) The denoising idea is to set zero for all coefficients that are less than thresholds.

(iii) The modified coefficients are used in an inverse transform to reconstruct the desired signal.

The thresholds in step (ii) are chosen according to Birge–Massart strategy (Birge and Massart 1997). The thresholds can be achieved as follows:

❑ Given the wavelet decomposition layer j, retain all the coefficients for $(j + 1)$ and higher layer.

❑ When the layer i satisfies the condition $1 \le i \le j$, keep n_i coefficients which have the largest absolute value; n_i is determined by: $n_j = M(j + 2 - i)^{\mathrm{ALPHA}}$. Normally, $\mathrm{ALPHA} = 3$ for denoising. A default value for M is $M = \mathrm{prod}(s(1,:))$ which is the number of low frequency coefficients. Parameter s is the wavelet decomposition structure of the seismic data to be denoised.

❑ Take the absolute coefficient value of i-th layer, and arrange the coefficient value in a sequence $|c(k_i)|$ by decreasing order; $c(k_i)$ is the k-th coefficient on this layer. The threshold λ_i can then be calculated as $\lambda_i = |c(n_i)|$.

4.1 Velocity model with horizontal layer

We first assume a velocity model shown in Fig. 2. The paper focuses on the random noise attenuation. According to the stochastic process theory, we can employ Gaussian random noise to illustrate the random noise. In order to verify the performance, seismic data with different signal-to-noise ratio (SNR) is provided.

As the wavelet denoising method is widely used in seismic signal processing with convincing performance, we apply the wavelet method for comparison to verify the performance of the method proposed. The parameters of vibratory seismic system used in simulation are: 50 ~ 200 Hz linear sweep signal (chirp signal), 0.001 s time step, 0.16 ms waveform length, a total of 49 seismic traces with spacing of 10 m. The refraction syncphase ax-

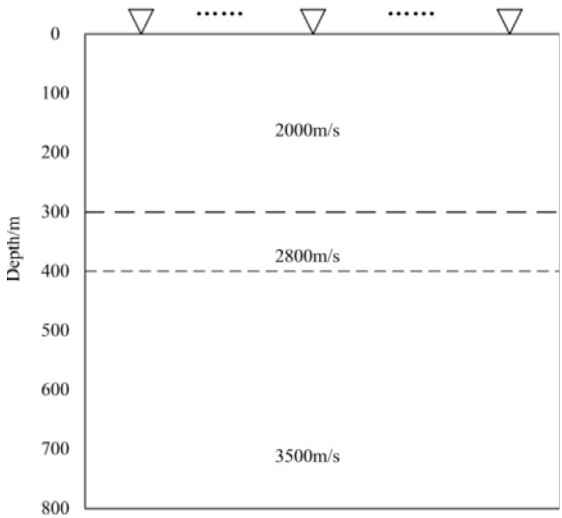

Fig. 2. Velocity model for numerical experiments.

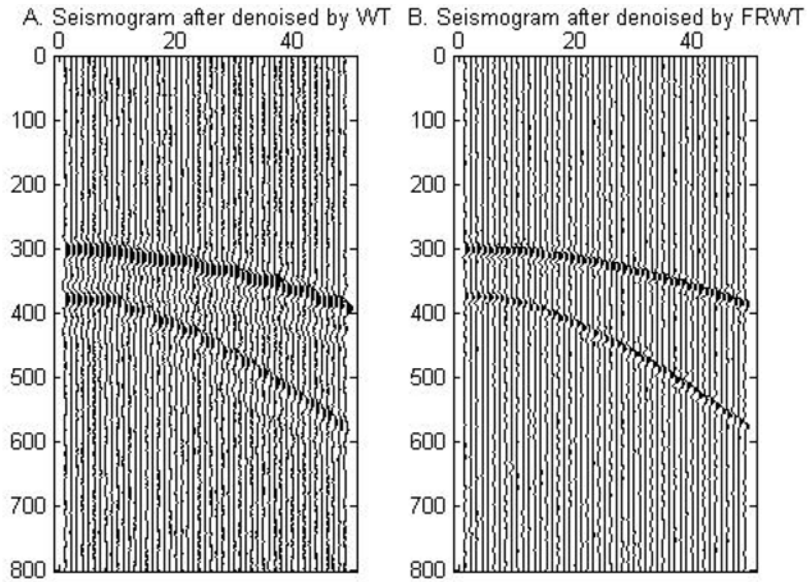

Fig. 3. Denosing performance comparison between WT and FRWT at SNR of 10 dB.

is from 300 and 400 m, respectively. We apply the WT and FRWT to denoise, as shown in Fig. 3 panels A and B, respectively. From both figures we can find that the noises are both removed effectively. The denoising per-

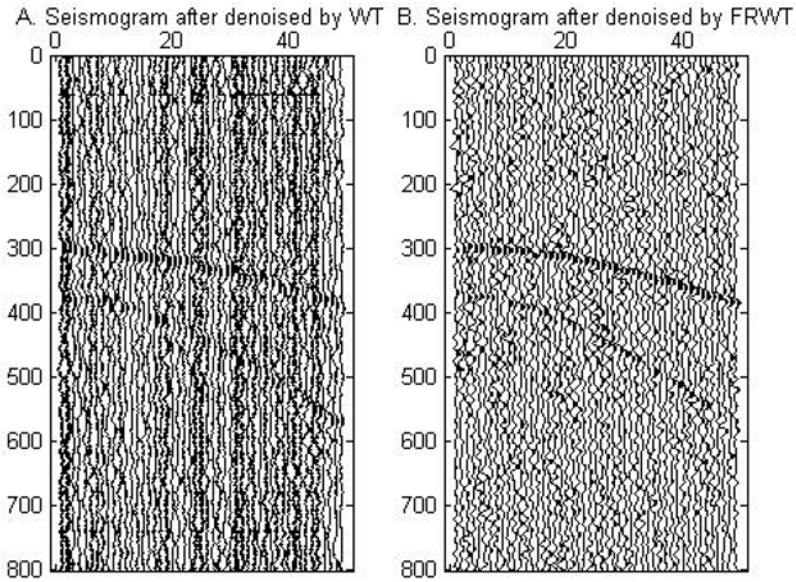

Fig. 4. Denoising performance comparison between WT and FRWT at SNR of –5 dB.

formance in Fig. 3B is better, since both quantity and continuity of syncphase axis are improved.

The parameters of the vibratory system used in Fig. 4 are the same as those in Fig. 3. We can find that with the increase of noise, the performance of the syncphase axis has decreased for both methods, WT and FRWT. However, the denoising performance in Fig. 4B efficient than that based on WT with the velocity model specified is still better than that in Fig. 4A. Thus, it is proved that the denoising method based on FRWT is more efficient than that based on WT with the velocity model specified.

4.2 Velocity model with inclined layer

To further analyse the performance of the proposed method, we also simulate a velocity model with an inclined layer. We assume a velocity model shown in Fig. 5. As in the previous model, only Gaussian random noise is addressed. Seismic data with different signal-to-noise ratio (SNR) is also provided.

The sampling parameters in Figs. 6 and 7 are the same as those in Figs. 3 and 4. The refraction syncphase axis at 400 m is inclined at 30 degrees. Comparing in Fig. 6 panel A with panel B, we can find that the denoising performance based on the FRWT method is better. Even at a SNR lower than 0 dB, the syncphase axis can be found with the FRWT method while it

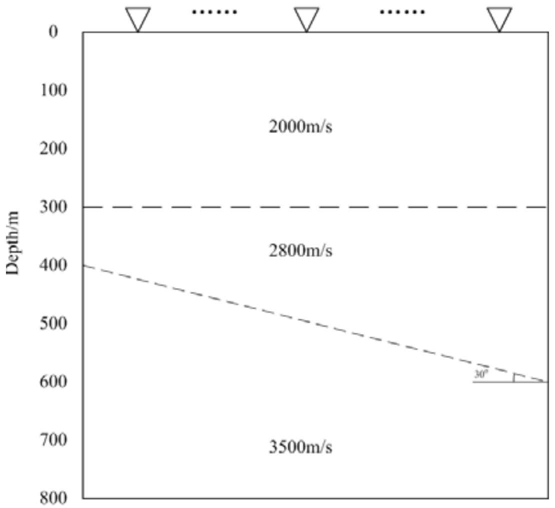

Fig. 5. Velocity model for numerical experiments.

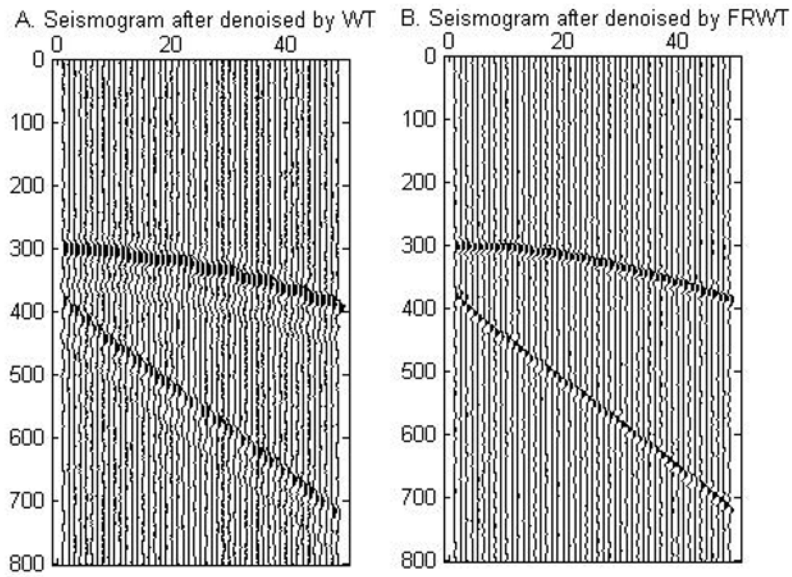

Fig. 6. Denoising performance comparison between WT and FRWT at SNR of 10 dB.

is not clear with WT method, as shown in Fig. 7. It is proved that the performance of denoising method based on FRWT is more efficient than that based WT method for the inclined layers with low SNR data.

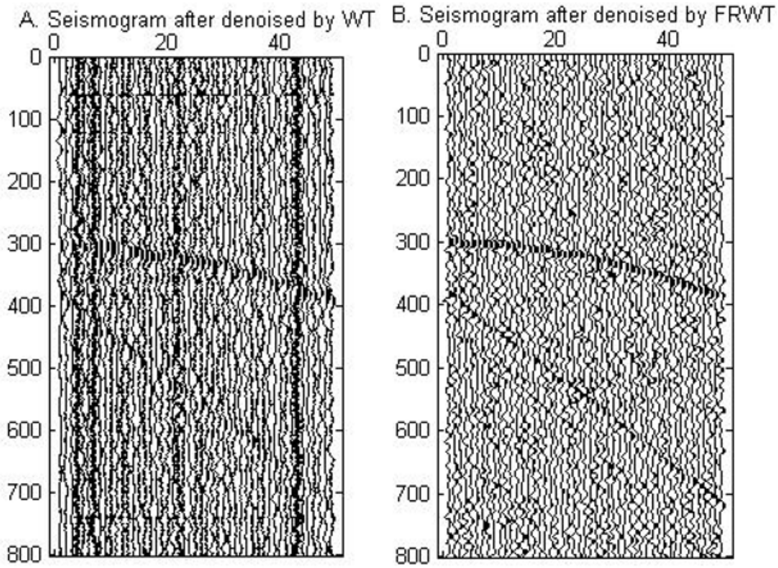

Fig. 7. Denoising performance comparison between WT and FRWT at SNR of –5 dB.

5. CONCLUSION

A novel method has been proposed for denoising seismic data in vibratory seismic systems based on the FRWT. This method can give better denoising performance, while the continuity of the syncphase axis in the seismic profile is maintained, which is very important to process the seismic data. The performance of the procedure is evaluated via simulation of seismic data with vibrator and pulse source. The results show that the FRWT is more efficient than traditional WT to suppress noise in vibratory seismic systems. Combining the special design of the seismic source, the quality and continuity of the syncphase axis in the seismic profile has been improved greatly.

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